

$$1) \lim_{n \rightarrow \infty} g(x) = \lim_{n \rightarrow \infty} 1 + \frac{n}{\sqrt{x^2+1}}$$

$$= \lim_{n \rightarrow \infty} 1 + \frac{n}{\sqrt{x^2(1+\frac{1}{x^2})}}$$

$$= \lim_{n \rightarrow \infty} 1 + \frac{n}{|x| \sqrt{1+\frac{1}{x^2}}}$$

$$= \lim_{n \rightarrow \infty} 1 + \frac{n}{-n \sqrt{1+\frac{1}{x^2}}} = \lim_{n \rightarrow \infty} 1 - \frac{1}{\sqrt{1+\frac{1}{x^2}}} = 0$$

$$\lim_{n \rightarrow \infty} g(x) = \lim_{n \rightarrow \infty} 1 + \frac{n}{\sqrt{x^2+1}}$$

$$= \lim_{n \rightarrow \infty} 1 + \frac{n}{\sqrt{x^2(1+\frac{1}{x^2})}}$$

$$= \lim_{n \rightarrow \infty} 1 + \frac{n}{n \sqrt{1+\frac{1}{x^2}}}$$

$$= \lim_{n \rightarrow \infty} 1 + \frac{1}{\sqrt{1+\frac{1}{x^2}}} = 2$$

$\lim_{n \rightarrow \infty} g(x) = 0 \Rightarrow \varphi_g$ admet un v.c. $y=0$
une Asy d'eq $y=0$

$\lim_{n \rightarrow \infty} g(x) = 2 \Rightarrow \varphi_g$ admet un v.c. $y=2$
une Asy d'eq $y=2$

b) pour tout $x \in \mathbb{R}$

$$g(x) = 1 + \frac{x}{\sqrt{x^2+1}}$$

$$g'(x) = \frac{x \sqrt{x^2+1} - x \sqrt{x^2+1}}{(\sqrt{x^2+1})^2}$$

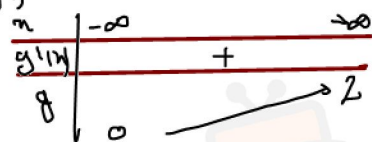
$$g'(x) = \frac{1 \sqrt{x^2+1} - x \frac{2x}{2 \sqrt{x^2+1}}}{(x^2+1)}$$

$$= \frac{\frac{\sqrt{x^2+1} \sqrt{x^2+1}}{\sqrt{x^2+1}} - \frac{x^2}{\sqrt{x^2+1}}}{(x^2+1)}$$



$$= \frac{x^2+1-x^2}{\sqrt{x^2+1}(x^2+1)} = \frac{1}{\sqrt{x^2+1}(x^2+1)}$$

1) a)



$$x \in]-\infty, +\infty[\Rightarrow 0 < g'(x) < 2$$

$$\Rightarrow g(x) > 0$$

2) $f(x) = x - 1 + \sqrt{x^2+1}$, $x \in \mathbb{R}$

a) Soient $x \in \mathbb{R}$

$$f'(x) = 1 + \frac{2x}{2\sqrt{x^2+1}} = 1 + \frac{x}{\sqrt{x^2+1}}$$

$$= g(x)$$

b) $\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} x - 1 + \sqrt{x^2+1}$

$$= \lim_{x \rightarrow -\infty} -1 + (\sqrt{x^2+1} + x)$$

$$= \lim_{x \rightarrow -\infty} -1 + \frac{(\sqrt{x^2+1} + x)(\sqrt{x^2+1} - x)}{(\sqrt{x^2+1} - x)}$$

$$= \lim_{x \rightarrow -\infty} -1 + \frac{x^2+1-x^2}{\sqrt{x^2+1} - x}$$

$$= \lim_{x \rightarrow -\infty} -1 + \frac{1}{\sqrt{x^2+1} - x} = -1$$

$$\Rightarrow \lim_{x \rightarrow -\infty} f(x) = -1$$

La droite d'éq $y = -1$ est asymptote à f au $U(-\infty)$

c) Pour tout réel x , $f'(x) = g(x)$
le signe de $f'(x)$ est celui de $g(x)$
d'après 1) d) $\forall x \in \mathbb{R}, g(x) > 0$

$$\Rightarrow f'(x) > 0$$

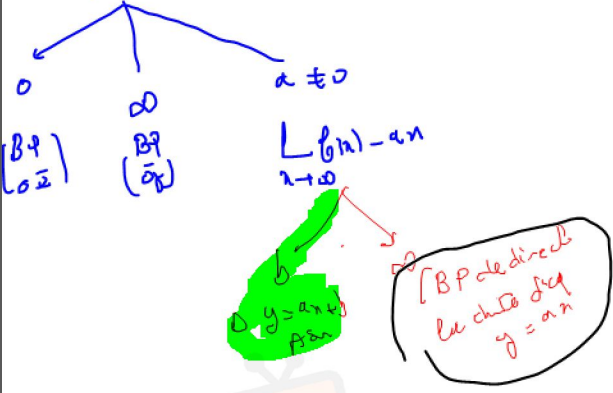


3a)

$$\begin{aligned} \lim_{n \rightarrow \infty} (f(n) - 2n + 1) &= \lim_{n \rightarrow \infty} n - 1 + \sqrt{n^2 + 1} - 2n + 1 \\ &= \lim_{n \rightarrow \infty} \sqrt{n^2 + 1} - n \\ &= \lim_{n \rightarrow \infty} \frac{(\sqrt{n^2 + 1} - n)(\sqrt{n^2 + 1} + n)}{\sqrt{n^2 + 1} + n} \\ &= \lim_{n \rightarrow \infty} \frac{n^2 + 1 - n^2}{\sqrt{n^2 + 1} + n} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 + 1} + n} = 0 \end{aligned}$$

$\lim_{n \rightarrow \infty} f(n) - 2n + 1 = 0$
 \Rightarrow La suite $D: y = 2n - 1$
 A-3) $a \in \mathbb{R}$ au $\forall n \in \mathbb{N}$

$$\lim_{n \rightarrow \infty} \frac{b(n)}{n}$$



$$\lim_{n \rightarrow \infty} f(n) - an > b$$

3b)

$$T: y = f'(0)(x - 0) + f(0)$$

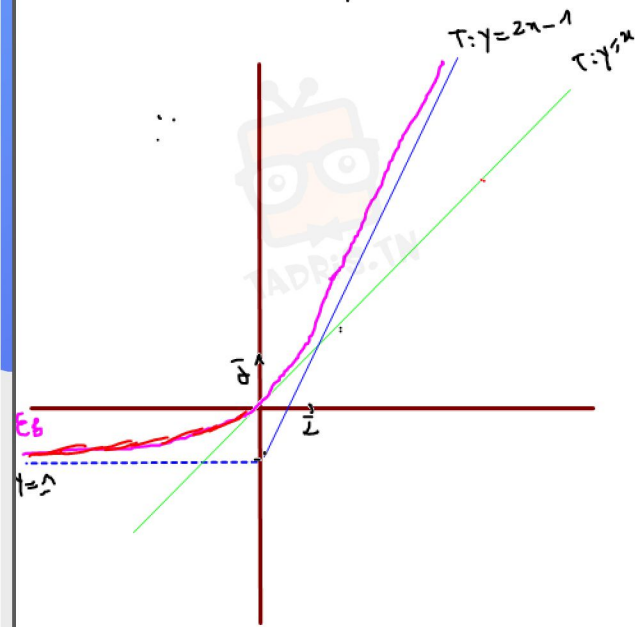
$$= 1(x - 0) + 0 = x$$

$$T: y = x$$

$$\begin{aligned} f(n) - y &= f(n) - n = n - 1 + \sqrt{n^2 + 1} - n \\ &= \sqrt{n^2 + 1} - 1 \end{aligned}$$



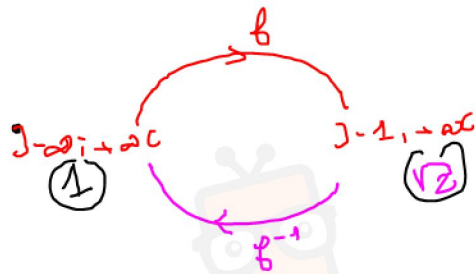
x	$-\infty$	0	$+\infty$
$f(x) - y$	$-$	0	$+$
point de def.	$e_{1/4}$	\wedge $e_{\text{conf}} +$	$e_{1/4}$



• f est strictement croissante

$\Rightarrow f$ réalise une bijection de \mathbb{R}

$$\text{sur } f(\mathbb{R}) =]-1, +\infty[$$



$$(f^{-1})(\sqrt{2}) = 1$$

$$(f^{-1})'(\sqrt{2}) = \frac{1}{f'(1)} = \frac{1}{1 + \frac{1}{\sqrt{2}}}$$

$$f:]-\infty, +\infty[\longrightarrow]-1, +\infty[$$

$$f(x) = x^2 + 1$$



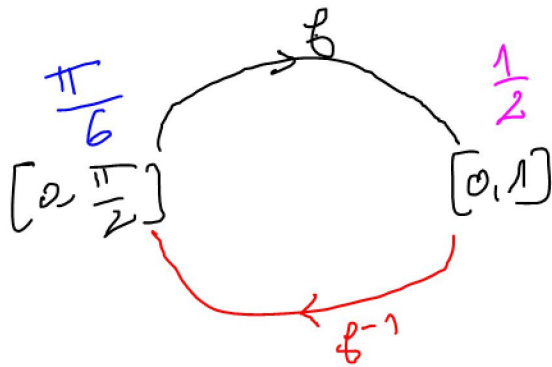
$$(f^{-1})'(5) =$$

$$f(x) = 5 \Leftrightarrow x^2 - 1 = 5$$

$$\Leftrightarrow x^2 = 6$$

$$\Leftrightarrow x = 2$$

$$(f^{-1})'(5) = \frac{1}{f'(2)} = \frac{1}{2x} = \frac{1}{4}$$



$$(f^{-1})'\left(\frac{1}{2}\right) = \frac{1}{f'\left(\frac{\pi}{6}\right)} = \frac{1}{\frac{\sqrt{3}}{2}}$$

$$= \frac{2}{\sqrt{3}}$$

